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| **Subject: METHODS MAT**  **Investigation 2, 2015**  **Topic – Arithmetic and geometric sequences**  **Take home component- SOLUTIONS** |  |

**Activity 1**

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| Maximum height = 4.178 m  Far left: height = 1 m  Far right: height = 1.457 m  Area = 4 x 25 = 100 m2 |

**Question 1 (a)**

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| The number of sheets required is 15 metres ÷ 20 cm i.e. sheets |

**Question 1 (b)**

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| Area of first metal sheet is |

**Question 1 (c)**

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| Area of each metal sheet is m2 |

**Question 1 (d)**

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| The expression  is the sum of a geometric sequence with *t1* = 1.10.2 with *r* = 1.10.2, and *n* =75 |

**Question 1 (e)**

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| Area = 0.1 x 15 m = 1.5 m2 |

**Question 1 (f)**

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| Determinethe sum of the 75 terms.      Total area = 33.655 + 1.5 = 35.155 m2 |

**Activity 2**

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| Width of strip is 20 cm. The function is  *n* = (25-15) ÷ 0.2 = 50 (50 sheets)  Using left hand values at *x* = 15, 15.2, 15.4, …., 24.8  Area above the ground  = 0.2×(20.3×0.915) + 0.2×(20.3×0.915.2) +0.2×(20.3×0.915.4) +…+0.2×(20.3×0.924.8)  = 0.2×20.3(0.915+0.915.2+0.915.4+….+0.924.8)  where 0.915+0.915.2+0.915.4+….+0.924.8 is a geometric series with *t1* = 0.9015, *r* = 0.90.2 and *n* = 50.    Area above the ground  = 0.2×20.3×  = 26.111 m2  The area under the ground = 50×0.1×0.2 = 1 m2.  Total area required for right side of the wall is 27.111 m2. |

**The Project**

**(a)**

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| Total area = 35.155 m2+27.111= 62.266 m2.  Volume of titanium required = 0.002×62.266 = 0.1245 m3.  Number of grams = 0.1245×1003×4.5  = 560250 grams  ≡ 560.25 kg  Minimum price at $15 kg is 560.25×$15 = $8403.75  Maximum price at $30 kg is 560.25×$15 = $16 807.50 |

**(b)**

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| The Council has $15 000 to spend on the memorial wall. The project seems feasible as the range of the cost varies from $8403.75 to $16 807.50.  The quantity of titanium required exceeds the minimum restriction. |

**(c)**

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| If both sides of the wall were covered in titanium, the minimum cost would be $16807.50. The project is not feasible as the cost is too high. |

**(d)** See solutions to Questions 1(d) and 1(f) and Activity 2

**Subject: METHODS MAT**

**Investigation 2, 2015**

**Topic – Arithmetic and geometric sequences**

**Validation- SOLUTIONS**

**Question 1 (a)**

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| Solution | Marking key/mathematical behaviours | Marks |
| (a) 1.80.2 = 1.125 units  (b) 0.2 x 1.80.2 = 0.225  (c) 5 ÷ 0.2 = 25  (d)  (e) | * Identifies 0.2 as the x value * Substitutes 0.2 into f(x) * Uses formula for rectangle * Calculates area * Multiplies or divides * Determines correct number * Expresses each term as an area * Forms a geometric sequence * Identifies correct values * Substitutes variables into formula for sum of terms * Determines sum of 25 terms | 1  1  1  1  1  1  1  1  1  2  1 |

**Question 2 (a)**

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| Solution  with | |
| Marking key/mathematical behaviours | Marks |
| * Includes the first term * Calculates each of the following four terms * Expresses terms in surd form where necessary | 1  2  1 |

**Question 2 (b)**

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| Solution  By Pythagoras theorem | |
| Marking key/mathematical behaviours | Marks |
| * Identifies the use of Pythagoras theorem * Provides a statement of the relationship of the sides | 1  1 |

**Question 2 (c)**

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| Solution  Pythagoras theorem: PX = | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes into Pythagoras theorem * Calculates the length of the hypotenuse | 1  1 |

**Question 2 (d)**

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| Solution            Construct a line, of length one unit, at right angles to the hypotenuse at point R. Join the end of the line to the central point to form the new right triangle. | |
| Marking key/mathematical behaviours | Marks |
| * Constructs lines at right angles to the hypotenuse * Draws lines of equal length * Labels diagram * Clarifies process used through description/ complete labelling | 1  1  1  1 |

**Question 2 (e)**

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| Solution  For an hypotenuse of 64 (642 = 4096) there will be 4095 triangles as the number of triangles is one less than the number of units for which the length of the hypotenuse is the square root. | |
| Marking key/mathematical behaviours | Marks |
| * Identifies relationship with square of 64 * Identifies number of triangles * Links number of triangles to square root of the hypotenuse * Determine number of triangles | 1  1  1  1 |

**Question 3 (a)**

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| Solution | Marking key/mathematical behaviours | | Marks |
| (i)  (ii)  (iii) | * Calculates first term * Calculates second term * Calculates third term | | 1  2  2 |
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**Question 3 (b)**

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| Solution | Marking key/mathematical behaviours | Marks |
| (i)    (ii) | * Copies pattern * Extends pattern   Identifies first term  States recursion | 1  1  1  1 |

**Question 3 (c)**

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| Solution | Marking key/mathematical behaviours | Marks |
| Terms are approaching 1.618  The fraction being added each time is decreasing in value | * Identifies oscillation * Identifies limiting number * Identifies decreasing amount to be added to each term | 1  1  1 |

**Question 2 (d)**

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| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Identifies two solutions * Expresses solutions in irrational form * Determines correct solutions for *α* and *β* | 1  1  1 |